Measurement uncertainty

Misconceptions and explanations

Contents

Test and measurement engineers have always been accompanied by the truth:

> "who measures, measures rubbish"

Now we can add:

> Measurement uncertainty is not being worried about being uncertain when measuring.
>
> The measurement uncertainty provides certainty with regard to the measurement result.
>
> Measurement uncertainty belongs to measurement like the hammer belongs to the carpenter.

Whereas measurement uncertainty was only a topic for sections of technical data sheets reserved for calibration laboratories and specialists just a few years ago, no measurement engineer can ignore it today.

The **GUM** (Evaluation of measurement data – **G**uide to the expression of **u**ncertainty in **m**easurement) with its documents

> JCGM 100:2008
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> JCGM 101:2008
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> JCGM 104:2009
>
> JCGM 200:2012

has established itself as a standard and has become standard reading for the measurement technician. Unfortunately there are also many possibilities to make mistakes in its application and interpretation. Some are listed here, together with comments.

Misconception: I always specify my measurement result with a numerical value and a physical unit, with which it is then complete.

The indication of the measurement uncertainty is still missing. Strictly speaking, a measurement uncertainty should be associated with every measured value. The GUM JCGM 100:2008 provides possible methods of representation in its paragraph "Reporting uncertainty".

For example, the measurement uncertainty is additionally given for the measured value I = 38.24 A. The standard measurement uncertainty is 0.10 A and the expanded measurement uncertainty 0.20 A with a coverage probability of 95 %. The index k = 2 refers to the expansion factor k, which in the case of normal distribution leads precisely to this 95 %.

The correct specification with expanded measurement uncertainty is:

> $I = 38.24$ A, $U_{0.95} = 0.20$ A
>
> $I = 38.24$ A ± 0.20 A, $\delta_s = 0.95$
>
> $I = 38.24$ A, $U_{k=2} = 0.20$ A
>
> I = (38.24 ± 0.20) A, where the expanded measurement uncertainty for the coverage interval 95 % is to be specified behind ±.

The standard measurement uncertainty is thus:

I = 38.24 A, uc = 0.10 A

I = 38.24 A, with standard measurement uncertainty uc = 0.10 A

I = (38.24 ± 0.10) A, where the standard measurement uncertainty is specified behind ±.

For good readability the measurement uncertainty should be displayed in the same unit as the measured value itself.

Misconception: My measured result is exact. I'm using an incremental encoder and it doesn't mis-count.

First of all: no measurement is exact. In addition: even if the incremental encoder never miscounts, there is still the inaccuracy of the start and end of the measurement, both of which can only be accurate to one line. At the ends there are each equally distributed reading errors of the span of a line width. Although these may be negligible in the case of a large number of lines, they are not zero.

Misconception: I've no time to determine the measurement uncertainty. It's too expensive and not worth the effort.

It cannot be denied that the determination of measurement uncertainty requires effort and (expensive) expertise. This applies to the filling-in of a measurement uncertainty budget and, to a lesser extent, if imc FAMOS is used to calculate the propagation of the measurement uncertainty.

The effort must be compared with the damage that arises when the measurement uncertainty is much larger than expected and the measurement result therefore differs too strongly from the true value. A tolerance may then be exceeded, data sheet specifications not met, etc.

Misconception: The expanded measurement uncertainty is double the standard measurement uncertainty.

There are conditions under which that is true, but not in general. The concepts of standard measurement uncertainty and expanded measurement uncertainty are defined quite differently:

Standard measurement uncertainty

Uncertainty of the measurement result, evaluation of deviations, expressed as a standard deviation. No probability is assigned to it.

Expanded measurement uncertainty

An interval around the measurement result in which a large proportion of the distribution of the values lies. The expanded measurement uncertainty is a coverage interval. The specification of the expanded measurement uncertainty is always accompanied by the specification of a coverage probability, e.g. 95 % or 99 %.

Definition according to GUM (JCGM 200:2012): product of the standard measurement uncertainty with a factor greater than one.

A relationship can be established between the two only with a known underlying distribution.

Expanded measurement uncertainty = k · standard measurement uncertainty

Where k is the expansion factor

For normal distribution this is

Expansion factor k	Probability
1	68.3 %
1.96 = approx.. 2	95 %
2.6	99 %
3	99.7 %

For uniform distribution and total width 2α:

$$\sigma = {}^{\alpha}/_{\sqrt{3}}$$, i.e.

Expansion factor k	Probability
1	57.7 %
1.73	100 %

For example, if the coverage probability is 95 %, then, following the above definition, it can be said: it is 95 % probable that the true value lies in the coverage interval. However, refer to the following paragraph for the interpretation.

Interpretation of coverage probability - coverage interval

Suppose the coverage probability is 95 %.

The direct application of the GUM leads to the statement: “The true value is included 95 % in the coverage interval” or “The true value is contained therein with 95 % probability”. But be careful in the interpretation!

The precise and detailed statement is: “In 95 % of all procedures or calculations in which a coverage interval is determined, the true value lies in the coverage interval”. So if, for example, the complete calculation of the expanded measurement uncertainty is carried out 100 times in imc FAMOS, the true value will lie in the determined coverage interval in approximately 95 of the cases. The 95 % says how reliable the estimation method is (i.e. the calculation in imc FAMOS in the example).

Incorrect statement: “The true value lies within the coverage interval, which has just been calculated in imc FAMOS and is now available, with a probability of 95 %”. The question of whether the current coverage interval contains the true value cannot be determined with a probability calculation, but only answered with yes or no.

The rolling of a die can be taken for a comparison. The probability of rolling a 1 is 1/6: if you roll 60 times, you will roll a 1 about 10 times. But if you have just rolled the die, the outcome of the random experiment is clear: it was a 3. In this case you can only determine that 3 is not 1, therefore no – and indeed 100 % no – for “no agreement”.

With this knowledge of the interpretation, it may then be said: “The true value lies 95 % in the interval”.

Misconception: The measurement uncertainty is 1K. So I have measured exactly to 1 K.

Without additional information we assume that the standard measurement uncertainty is meant. The standard measurement uncertainty is the standard deviation σ of the measured value, if the measured value is taken as a random variable. A confidence interval [-σ, σ] includes the true value only in about 68 % of all random experiments on the basis of normal distribution. In the example, σ is 1K. Thus, the true value lies within an interval [-1K, 1K] around the measured value in only about 2/3 of all measurements. Hence outside in 1/3 of all measurements.

The standard measurement uncertainty represents above all a quality parameter in this context.

If one is interested in the area in which the measured values almost always lie, then a multiple needs to be used.

The above interval increases to [-k·σ, k·σ]. See above for the definition of the expansion factor k. Calibration laboratories frequently work with 95 % certainty. If one wishes to be almost completely sure, k = 3 is selected. This then leads to an interval [-3K, 3K]. It is as though one had measured to ±3K exactly.

This multiple is the expanded measurement uncertainty, by the way – see above.

It should not be denied here that in many cases small numbers are preferred so that deviations appear small. The standard measurement uncertainty is very welcome in this case. To get a good feeling for the real possible deviations, a factor of 2 or 3 is still to be applied. Caution: these factors apply exactly only with a normal distribution. Those who have no additional knowledge, however, assume normal distribution.

Misconception: I've measured 10 times, taken the mean value and the result is now 10 times as good or accurate.

The measurement result has a standard deviation which can already be regarded as a measure of the accuracy. However, the standard deviation is not the accuracy. If several measured values are now averaged, the standard deviation decreases with $\sqrt{N}$:

$$\bar{u} = \frac{u}{\sqrt{N}}$$

where

u is the standard deviation of the individual measurement

$\bar{u}$ is the standard deviation of the averaged measurement

N is the number of repetitions

The standard deviation decreases by a factor of 10 only when 100 measured values are averaged. One can then say that the result is 10 times as good or accurate.

Misconception The measurement uncertainty is 1.2345V.

The measurement uncertainty can be determined and secured experimentally with some (occasionally great) effort. Otherwise, it is determined from data sheets for the components involved in the measurement by filling in and calculating a measurement uncertainty budget. One's own considerations and estimations are also incorporated. However, the data in the data sheets used were in turn created in the same way. The mathematical framework for the determination of measurement uncertainty according to GUM is exact and the calculation rule is clear, but the numerical values flowing into it are unfortunately often inaccurate.

Generally, no high accuracy can be attributed to a measurement uncertainty. A value with 1 or 2 decimal places is generally realistically achievable. Anything else is an exception. The measurement uncertainty should therefore be specified only with so many decimal places as are useful, even after precise calculation. A supposedly high precision just appears unbelievable.

Example: A sensor and an amplifier each have a measurement uncertainty of u_1, u_2 = 1V. The combined uncertainty u is given by

$$u = \sqrt{u_1^2 + u_2^2}$$

i.e. u = 1.414V. Precisely calculated to many decimal places. However, one may assume that, for example, u1 = 1V is itself certainly not exact to 3 decimal places, i.e. also not the calculation result. A specification of

u = 1.4V

is certainly appropriate.

Misconception: My measurement setup and evaluation is so complex that I cannot determine any measurement uncertainty.

The more complex the measurement setup, the more important the determination of the measurement uncertainty becomes. The GUM provides numerous hints and examples of how to determine the measurement uncertainty budget.

It is permissible to carry out several test measurements. A good indication can be obtained by determining the standard deviation of the scattered measurement results.

One can look at the noise on a signal. If the noise is not really a fluctuation of the real value, but has other causes, then that already makes a contribution to the measurement uncertainty. The standard deviation of the noise band provides a good indication.

Sometimes one has to estimate, e.g. from experience.

If an end result is to be achieved from the raw values by calculation, then the propagation of the measurement uncertainty can be determined by calculation with imc FAMOS.

Misconception: imc FAMOS is too imprecise in determining the measurement uncertainty.

imc FAMOS determines the measurement uncertainty by means of the Monte Carlo method. The accuracy achieved depends on the number of Monte Carlo tests and can be increased with a higher number. Nevertheless, it is practically impossible to determine the measurement uncertainty with 7 valid digits. But in any case (see above) that is never necessary.

In many simple calculation cases, such as the multiplication of two measured variables for example, a precise solution can be found very easily with the help of the GUM uncertainty framework. The solution is really precise, which is not achievable using the Monte Carlo technique. But which, as already stressed, is also not necessary.

However, there are many evaluations, e.g. using non-linear processing, where the equations of the GUM in the Taylor expansion only represent approximations. Despite great mathematical effort using linearisation, etc., one only obtains an estimate. And that can be really wrong in the case of greater non-linearity. At this point the Monte Carlo method is actually (significantly) more accurate.

At this point, where it comes to accuracy, one doesn't need to mention the number of algorithms where a calculation strictly according to GUM uncertainty framework is practically unfeasible, for instance FFT and digital filtering.

Misconception: My evaluation algorithm is robust, as I have proven on the basis of my (single) measurement.

There are only a few evaluation algorithms whose correctness can be proven on the basis of a single measurement. It is better to check the algorithm by taking many measurements. If not many measurements are available, however, then the robustness of the algorithm can be verified in imc FAMOS with the aid of assumed measurement uncertainties of the input data by performing a measurement uncertainty calculation. The method can be improved by checking the intermediate and final results using the available snapshot functions.

Misconception: In the data sheet for a component, it says: "Typical deviations 1 mV". This is the standard deviation.

In data sheets one frequently finds the following specifications, whose relationship to the measurement uncertainty is explained:

Specification in the technical data sheet	Interpretation
Typical deviations	The exact meaning is not defined. One cannot even say that a manufacturer has really clearly defined it for itself. Even if there is a definition, it is generally not known to the user. The likely assumption is that the deviations, which of course generally do not exhibit precisely this typical value, are scattered in a range in which a large percentage of the components or measurements is covered. It remains unclear whether this percentage is closer to 60 %, 90 % or even higher. Hence, it also cannot be said whether this data sheet specification corresponds to the measurement uncertainty or even a multiple (about 2 or 3 times) of the measurement uncertainty.
Maximum deviation	This is an error limit. Error limits are maximum values and represent guaranteed properties. They have nothing to do with the measurement uncertainty, which should normally be much smaller than an error limit.
Typical gain uncertainty	Here, the word uncertainty is obviously being used for deviation. See typical deviations, but applied to the gain factor.

Maximum uncertainty of the reference point	Here, the word uncertainty is obviously being used for deviation. See maximum deviations, but applied to the reference point. The measurement uncertainty is a fixed value, it can neither be typical nor maximum.
Measurement uncertainty < ±1K	The measurement uncertainty is always defined as a positive fixed value. Only measurement uncertainty = 1 K can be meant here.

Additional information:

imc Test & Measurement GmbH
Voltastr. 5
13355 Berlin, Germany

Telephone:	+49 (0)30-46 7090-0
Fax:	+49 (0)30-46 31 576
E-mail:	hotline@imc-tm.de
Internet:	http://www.imc-tm.com

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